Endogenous Stackelberg leader-follower relations ⋆

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Abstract: We study the endogenous Stackelberg relations in a dynamic market. We analyze a twice-repeated duopoly where, in the beginning, each firm chooses either a quantity-sticky production mode or a quantity-flexible production mode. The size of the market becomes observable after the first period. In the second period, a firm can adjust its quantity if, and only if, it has adopted the flexible mode. Hence, if one firm chooses the sticky mode whilst the other chooses the flexible mode, then they respectively play the roles of a Stackelberg leader and a Stackelberg follower in the second marketing period. We compute the supply quantities at equilibrium and the corresponding expected profits of the firms. We also analyze the effect of the slope parameter of the demand curve on the expected supply quantities and on the expected profits.

Keywords: Game Theory, Stackelberg games, equilibrium, uncertainty.

1. INTRODUCTION

Stackelberg leader-follower relations have most often been modeled in association with the chronological order of moves. Namely, there are a first mover (leader) and a second mover (follower). In spite of such a supposedly dynamic setting, it has been common to overlook what happens during the period between these two moves, by assuming a static market which clears only once, after the second mover's move. This builds certain biases into the analysis of firms' strategic incentives either to lead or to follow, which are the contributing forces to endogenous Stackelberg outcomes.

In the earlier literature, endogenous leader/followership has been imbedded most often in the context of a timing game played by oligopolists. Hamilton and Slutsky (1990) construct an 'extended game' framework, in which each firm faces the choice of production timing. A fair number of theoretical explanations have been attempted with regard to firms' incentives for Stackelberg behavior, especially a follower's incentive to wait. Robson (1990) imposes costs associated with an early action. Albaek (1990) takes into account cost uncertainty. The effect of *a priori* informational heterogeneity between firms, broadly defined, have been discussed in several studies, including Mailth (1993) and Normann (1997). On the other hand, when the oligopolists are *a priori* equally uncertain about the market demand, as in Spencer and Brander (1992), Sadanand and Sadanand (1996) and Maggi (1996), earlier production can utilize less information in exchange for the strategic advantage of commitment, whereas later production does the converse. Hirokawa and Sasaki (2001) employed a similar framework to Hamilton and Slutsky's 'extended game', except that the static market is replaced with an explicitly two-period market. This can be carried out in two alternative ways. One is to interpret firms' moves as their entry timing. This means that, if there is only one 'first mover', then it becomes a monopolist in its first production period. Hirokawa and Sasaki (1998b) adopts this interpretation. The other alternative is to assume that, at the beginning of the game, firms are already operating in the market. That is, even a 'second mover' is producing in the first marketing period as well as in the second marketing period. This is the promise we adopt in this paper (as Hirokawa and Sasaki (2001) did).

In Hirokawa and Sasaki's model, firms have to choose between a quantity-flexible production mode and a quantity-sticky production mode, the latter implying that the firm's production quantity be unchanged between the two periods. [1] Quantity stickiness entails two effects. On one hand, it serves as a device for quantity commitment throughout the two periods. This is a positive, deterministic effect. On the other hand, it hinders the firm's flexibility in adjusting its supply quantity once demand uncertainty resolves in the second period. This is a negative, stochastic effect. Under some conditions, the tradeoff between these two effects can give rise to *a posteriori* asymmetric Stackelberg-like behaviour, even if the firms are *a priori* identical. Firms' payoffs consists not only of the Stackelberg leader's

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[1] Quantity stickiness can arise from various sources, including technology, precommitted capacity, advance production and inventory investment, binding contracts, and so on.

and follower's profits in the second marketing period, but also of the pre-Stackelberg profits in the first period.

In this paper we consider a more general inverse demand function in the model of Hirokawa and Sasaki (2001), and we compute the outputs at equilibrium.

2. THE MODEL AND THE EQUILIBRIUM

Two *a priori* identical firms operate in the same industry. The game lasts through two marketing periods. At the beginning of the game, each firm chooses between a quantity-flexible production mode and a quantity-sticky production mode. If a firm chooses the stick mode, then the firm must supply the same quantity in both periods. This restriction dos not apply if the firm chooses the flexible production mode in the beginning. We assume that quantity stickiness becomes mutually observable before each firm sets its supply quantity.

For simplicity, the two firms are assumed to sell homogeneous products (i.e. perfect substitutes). The inverse demand function in each marketing period is $P = A - bQ$, where $Q = q_1 + q_2$ is the sum of the two firms' outputs. The demand intercept A is *ex ante* stochastic, of which the prior cumulative distribution $F(a)$ is commonly known to the two firms, with finite mean $E(A) > 0$ and variance $V(A)$. This intercept stays unchanged throughout the two marketing periods. The parameter $b \geq 1$ is the slope parameter of the demand curve. We consider prices net of marginal costs. The discount factor is δ, where $0 < \delta \leq 1$. To ensure individual rationality, we assume

$$\inf(A) \geq \frac{1+\delta}{3+2\delta} E(A) \tag{1}$$

$$\inf(A) \geq \frac{4+3\delta}{2(3+2\delta)} E(A). \tag{2}$$

The intercept A is unobservable to the firms until the price and quantities are realized at the end of the first period. Then, once the state of demand has been observed, a firm can make use of this information to optimize its second-period supply quantity if, and only if, the firm has chosen the flexible mode in the beginning. On the other hand,if only one of the firms has selected the sticky mode whilst the other has selected the flexible mode, then the stickiness entitles the firm to Stackelberg leadership in the second marketing period. Therefore, at the beginning of the game, firms face the tradeoff between the strategic advantage of commitment and the adjustability to the demand realization.

The profit maximization problems for each firm F_i, with $i \in \{1,2\}$, are as follows, depending upon the two firms' commitment decisions: $q_i^{I|X}$, $q_i^{II|X}(A)$ and $q_i^{L|X}$ denote an uncommitted (quantity-flexible) firm's quantity in the first and the second marketing stages, and a committed (quantity-sticky) firm's quantity throughout the game, respectively, where X indicates the opponent firm's quantity stickiness: $X = F$ if the opponent is uncommitted, or $X = L$ if committed. Note that only an uncommitted firm's second-period quantity can be made contingent upon the state A.

1) If both firms commit:

$$\max_{q_i^{L|L}} E\left(\left(A - bq_1^{L|L} - bq_2^{L|L}\right) q_i^{L|L}\right), \quad i = 1, 2.$$

2) If firm F_i commits whilst the other firm F_j does not, then the game is solved backwards. In the second marketing stage, firm F_j solves:

$$\max_{q_j^{II|L}(A)} \left(\left(A - bq_i^{L|F} - bq_j^{II|L}(A)\right) q_j^{II|L}(A)\right).$$

Let $q_j^{II}(A, q_i^{L|F})$ denote the solution for this maximization. Back in the first stage, the two firms solve, respectively:

$$\max_{q_i^{L|F}} E\left(\left(A - bq_i^{L|F} - bq_j^{I|L}\right) q_i^{L|F} + \right.$$

$$\left. + \delta\left(A - bq_i^{L|F} - bq_j^{II}(A, q_i^{L|F})\right) q_i^{L|F}\right),$$

$$\max_{q_j^{I|L}} E\left(\left(A - bq_i^{L|F} - bq_j^{I|L}\right) q_j^{I|L}\right).$$

3) If neither firm commits, in the second stage:

$$\max_{q_i^{II|F}(A)} \left(\left(A - bq_1^{II|F}(A) - bq_2^{II|F}(A)\right) q_i^{II|F}(A)\right),$$

with $i = 1, 2$,

and back in the first marketing stage:

$$\max_{q_i^{I|F}} E\left(\left(A - bq_1^{I|F} - bq_2^{I|F}\right) q_i^{I|F}\right), \quad i = 1, 2.$$

The game specified above can be summarized into the following payoff matrix:

Table 1. Payoff matrix

Firm	Opponent: Commit	Opponent: Not commit
Commit (quantity-sticky)	$\pi^{L\|L}$	$\pi^{L\|F}$
Not commit (quantity-flexible)	$\pi^{F\|L}$	$\pi^{F\|F}$

Theorem 1. (i) In Cournot-Nash outcomes (i.e. outcomes 1) and 3)), each firm's expected supply quantities are:

$$q_i^{L|L} = q_i^{I|F} = \frac{E(A)}{3b}, \qquad q_i^{II|F}(A) = \frac{A}{3b},$$

with $i \in \{1,2\}$.

(ii) In a Stackelberg-like outcome, the committed firm's supply quantity is:

$$q_i^{L|F} = \frac{1+\delta}{b(3+2\delta)} E(A),$$

whilst the uncommitted firm's supply quantities are:

$$q_i^{I|L} = \frac{2+\delta}{2b(3+2\delta)} E(A),$$

$$q_i^{II|L}(A) = \frac{A}{2b} - \frac{1+\delta}{2b(3+2\delta)} E(A),$$

with $i \in \{1,2\}$.

Proof. The results follow by solving optimization problems 1)-3), assuming that positivity constraints are unbinding. □

From Theorem 1, we get the following corollaries:

Corollary 2. (i) In Cournot-Nash outcomes, each firm's expected supply quantities do not depend upon the discount factor δ.

(ii) In a Stackelberg-like outcome, the committed firm's supply quantity increases in δ, whilst the uncommitted firm's supply quantities decrease in δ.

Corollary 3. Each firm's expected supply quantities decrease in the slope parameter b of the demand curve.

Corollary 4. The expected profits of the firms are as follows:

$$\overline{\pi}^{L|L} = (1+\delta)\frac{E(A)^2}{9b},$$

$$\overline{\pi}^{L|F} = \frac{2+\delta}{2b}\left(\frac{1+\delta}{3+2\delta}E(A)\right)^2,$$

$$\overline{\pi}^{F|L} = \left(1+\frac{\delta}{b}\right)\left(\frac{2+\delta}{2(3+2\delta)}E(A)\right)^2 + \frac{\delta}{4b}V(A),$$

$$\overline{\pi}^{F|F} = \left(1+\frac{\delta}{b}\right)\left(\frac{E(A)}{3}\right)^2 + \frac{\delta}{9b}V(A)$$

when, and only when, conditions (1) and (2) are met.

3. CONCLUSIONS

We have analyzed endogenous Stackelberg-like behavior in light of quantity stickiness in an explicitly dynamic market. We computed the supply quantities at equilibrium and the corresponding expected profits of the firms. We saw that each firm's expected supply quantities and the corresponding firms' expected profits decrease as the slope parameter b of the demand curve increases.

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